

# ANALYTICAL EXPRESSIONS OF TRANSIENT THERMAL RESPONSE OF SELF-HEATING SEMICONDUCTOR DEVICES

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## ABSTRACT

Analytical expressions of the transient thermal response of self-heating devices have been derived based on a feedback amplifier circuit model, yielding accurate methods to extract the thermal time constant in both the time and frequency domains. The transient thermal responses of GaInP/GaAs HBT were measured and explained using the expressions derived.

## INTRODUCTION

The transient thermal effect must be accounted for if the high-frequency and switching performances of self-heating semiconductor devices are to be accurately modeled.[1-2] This applies to AlGaAs/GaAs and GaInP/GaAs HBTs, which have been widely used to fabricate microwave power amplifiers, where the lower thermal conductivity of the GaAs substrate enhances the self-heating.

The self-heating effect, caused by the temperature rise due to the power dissipation and the temperature dependence of device performance, can be regarded as a feedback inside the device.[3] The thermal feedback can either be negative or positive depending on the kind of device and the operation state of the device. In the case of an HBT with emitter grounded, for example, the thermal feedback is negative (positive) when a constant base current (voltage) is applied,[4] and the amount of the thermal feedback is dependent on the dissipated power.

Previously, physical models and circuit models have been proposed to describe the transient thermal response of self-heating devices. The physical models, based on the simultaneous calculation of the

current and heat flow inside devices,[5,6] lead, however, to either complicated expressions or numerical results, and thus are not suitable for circuit design and device characterization. The circuit model, proposed by Baureis et al,[7] and modified by Suh et al,[8] is available only for the HBTs under a specific operation state. In addition, both the physical and circuit models mentioned above can be used only in the time domain.

In the present work, explicit analytical expressions are derived based on a circuit model proposed. Our results describe, for the first time, 1) the transient thermal response of devices such as BJTs, HBTs, MOSFETs, MESFETs and HEMTs, 2) the response in both thermal stable and unstable states, and 3) the response due to both the negative and positive thermal feedback, and can be used to extract thermal time constant in both the time and frequency domains.

## DERIVATION

A feedback amplifier circuit model is used to represent the self-heating device in this study. The model is composed of a linear basic amplifier and a

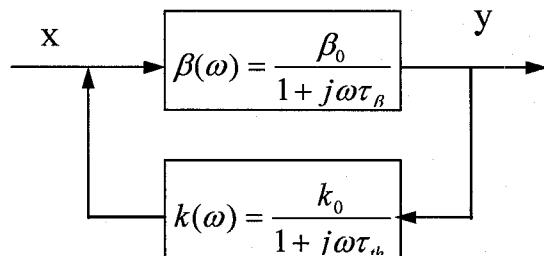


Fig. 1. Block diagram of feedback amplifier model of self-heating device.

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linear feedback circuits as shown in Fig. 1.

The basic amplifier represents the isothermal performance of the device (e.g. BJT, HBT, MOSFET, MESFET, or HEMT), and can be expressed in frequency domain as,[9]

$$\beta(\omega) = \beta_0 / (1 + j\omega\tau_\beta), \quad (1)$$

where  $\beta_0 = \frac{\partial y}{\partial x} \Big|_{\omega=0}$ , x and y are the input and output

of the device, respectively, and  $1 / \tau_\beta$  corresponds to the electrical cutoff frequency. The feedback circuit represents the temperature rise due to the power dissipation and the temperature dependence of the device performance, and can be expressed as [10]

$$k(\omega) = k_0 / (1 + j\omega\tau_{th}), \quad (2)$$

where  $k_0 = \frac{\partial x}{\partial T} \frac{\partial T}{\partial y} \Big|_{\omega=0}$ , T is the temperature inside

the device,  $\tau_{th} = \sqrt{RC}$  is the thermal time constant, R and C are the thermal resistance and thermal capacitance, respectively.

The frequency response of the self-heating device, namely the close loop gain of the feedback amplifier,

$$\frac{\beta(\omega)}{1 - \beta(\omega)k(\omega)}, \quad \text{can be simplified as}$$

$$G(\omega) = \frac{\beta_0}{1 - f / (1 + j\omega\tau_{th})}$$

$$= \begin{cases} \beta_0 \{1 + \frac{f}{1-f} \frac{1}{[1 + j\omega\tau_{th} / (1-f)]}\} & \text{if } f \neq 1 \\ \beta_0 (1 + \frac{1}{j\omega\tau_{th}}) & \text{if } f = 1 \end{cases} \quad (3)$$

$$= \begin{cases} \beta_0 (1 + \frac{1}{j\omega\tau_{th}}) & \text{if } f = 1 \end{cases} \quad (4)$$

as  $\tau_{th} \gg \tau_\beta$  holds for BJTs, HBTs, MOSFETs, MESFETs, and HEMTs. Here

$f = \beta_0 k_0 = \frac{\partial y}{\partial x} \frac{\partial x}{\partial T} \frac{\partial T}{\partial y} \Big|_{\omega=0}$  is the loop gain of feedback amplifier.

When  $f < 1$ , Eq. (3) can be rewritten as

$$G(\omega) = G_h + (G_l - G_h) \frac{1}{[1 + j\omega\tau_{th}(G_l / G_h)]}, \quad (5)$$

where  $G_l = \frac{\beta_0}{1-f}$  ( $G_h = \beta_0$ ) is the gain of the

device in the frequency range of  $\omega \ll 1 / \tau_{th}$

( $1 / \tau_{th} \ll \omega \ll 1 / \tau_\beta$ ) by considering (neglecting)

the self-heating. Since the frequency at the 3dB point of the frequency response does not correspond directly to  $\tau_{th}$  as shown in Eq. (5), the method to extract  $\tau_{th}$  in frequency domain proposed by Bruce et al [11] should thus be modified.

The output of self-heating device in response to a step input can be obtained from the inverse Laplace transform [12] of Eqs. (3) and (4) as

$$y(t) = \begin{cases} \beta_0 x \left\{ 1 + \frac{f}{1-f} \left[ 1 - \exp\left(-\frac{t}{\tau_{th} / (1-f)}\right) \right] \right\} & \text{if } f \neq 1 \\ \beta_0 x (1 + t / \tau_{th}) & \text{if } f = 1 \end{cases} \quad (6)$$

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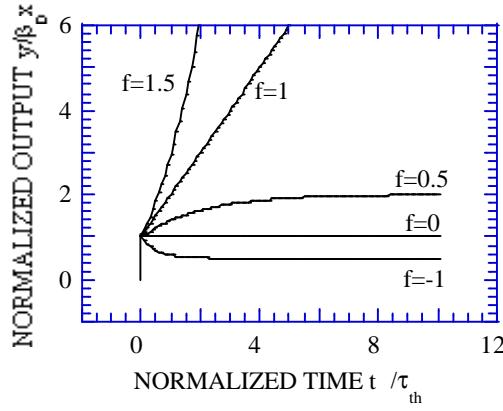


Fig. 2. Calculated transient thermal response of self-heating devices.

The transient thermal responses calculated using Eqs. (6) and (7) are shown in Fig. 2. The device is stable when  $f < 1$ , the output decreases (increases) with time during the transient and the response become faster (slower) with the increasing the amount of the negative (positive) feedback. The device become unstable and may be destroyed by thermal runaway when  $f \geq 1$ , and the output increases linearly (exponentially) with time for  $f=1$  ( $f>1$ ).

When the device is stable ( $f < 1$ ), Eq. (6) can be rewritten as

$$y(t) = y_i + (y_s - y_i) \left\{ 1 - \exp\left[-\frac{t}{\tau_{th} (y_s / y_i)}\right] \right\} \quad (8)$$

$y_i = \beta_0 x$  ( $y_s = \frac{\beta_0 x}{1-f}$ ) in Eq. (8) is the output at the

initial (steady) state of the thermal transient, where the

self-heating effect is negligible (strong). Taking the exponential factor of  $y(t) \propto [1 - \exp(-t / \tau)]$  as

$\tau_{th}$  [7] may underestimate (overestimate) the thermal time constant for negative (positive) thermal feedback. An expression similar to Eq. (8) was

derived by Suh et al in time domain for HBTs with negative feedback.[8]

## EXTRACTION

Using the setup shown in Fig. 3, the transient thermal response measurement was performed on GaInP/GaAs HBT with emitter size of  $2.4 \times 20 \mu\text{m}^2$ .

The transient thermal response of the HBTs under emitter-base voltage pulse and dc collector-emitter voltage are shown in Fig. 4, the dots are measured responses and the lines are the fitting curves using Eq. 8. Because of the positive thermal feedback, an increase in the collector current with time during the transient can be observed. Figure 5 shows the dissipated power dependence of  $\tau$  extracted using

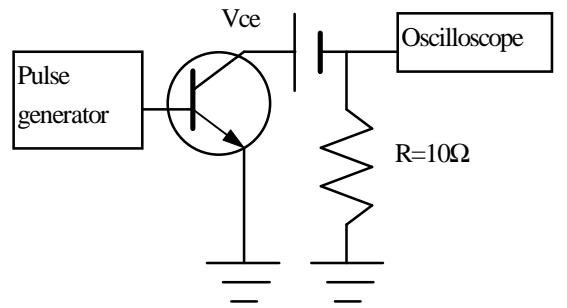


Fig. 3. Setup for measuring transient thermal response.

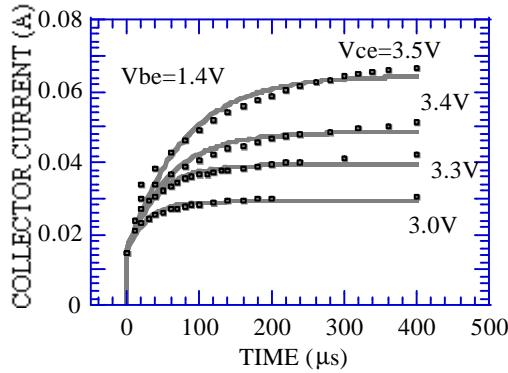


Fig. 4. Measured thermal transient response of an HBT under pulsed  $V_{be}$  bias operation.

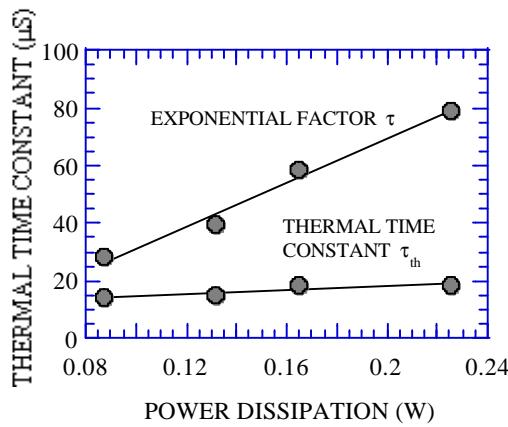


Fig. 5. Extracted thermal time constant versus power dissipation.

$y(t) \propto [1 - \exp(-t / \tau)]$ , which can be regarded as a figure of merit of the response speed. The increase in  $\tau$  with increasing dissipated power, that is the slowing of the thermal response with increasing the positive feedback, was confirmed experimentally for the first time. Superimposed on the plot are the thermal time constants extracted using Eq. (8), an average value of  $\tau_{th} = 16\mu s$  was obtained. The

differences between  $\tau$  and  $\tau_{th}$  in Fig. 5 indicate the error in extracting thermal time constant using the conventional method. [7]

## CONCLUSION

Analytical expressions of transient thermal response of self-heating devices have been derived, which can be used to extract thermal time constant and predicate the thermal stability of self-heating devices in both the frequency and time domains. The dependence of the thermal response speed on the thermal feedback was confirmed and the thermal time constant  $\tau_{th} = 16\mu s$  was extracted for  $2.4 \times 20\mu m^2$  GaInP/GaAs HBTs.

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